# Entanglement generation via a completely mixed nuclear spin bath

H. Christ, J. I. Cirac, and G. Giedke
*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*
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We show that qubits coupled sequentially to a mesoscopic static completely mixed spin bath via the Heisenberg interaction can become highly entangled. Straightforward protocols for the generation of multipartite entangled Greenberger-Horne-Zeilinger (GHZ) states are presented. We show the feasibility of an experimental realization in a quantum dot by the hyperfine interaction of an electron with the nuclear spins.

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## I. INTRODUCTION

The quest to realize quantum information processing (QIP) has motivated an impressive race to implement high-precision preparation and manipulation of isolated two-level quantum systems (qubits) in a wide variety of physical settings.[1] A hallmark achievement for each such approach is the generation of quantum entanglement through controlled interaction between two or more qubits. Since switchable direct interactions between qubits often entail additional decoherence mechanisms, many QIP proposals rely on interactions *mediated* by an additional quantum system. As a rule this mediator (just as the qubits themselves) needs to be prepared in a pure state to achieve high-fidelity quantum operations, and it may look futile to use a high-entropy mesoscopic spin bath for this task. In contrast to these expectations, we show here that high-fidelity entanglement generation can be realized even if the qubits can interact only with an arbitrarily mixed spin bath, provided that this interaction can be switched on and off, single-qubit unitaries are available, and the bath has slow internal dynamics. This is motivated by and will be illustrated through the example of electron spin qubits in quantum dots (QDs),[2] where the ensemble of lattice nuclear spins represents a strongly coupled but slowly evolving spin bath.

Nuclear spins in quantum dots have received much theoretical[3–8] and experimental[9,10] attention in the QIP context as the main source of electron spin decoherence through the strong hyperfine coupling. It has also been noted that their slow internal dynamics and long (expected) decoherence time[11] make the ensemble of nuclear spins useful as a quantum memory[12] or for quantum computation.[13] These applications, however, require careful yet unachieved preparation of the nuclear system. What we show here is that the unprepared highly or even maximally mixed (nuclear) system is able to mediate coherent interaction between electrons and thereby allows the generation of highly entangled states of many (electron spin) qubits without any electron-electron interaction.

We consider a QD in the single-electron regime[14] and assume the availability of single-electron state preparation and measurement as well as the controlled shuttling of prepared electrons into and out of the QD, all of which have been demonstrated experimentally.[15] Additionally required is control of the detuning (e.g., by a magnetic or electric field), which switches the hyperfine (HF) interaction between resonant and off-resonant regimes. We first show how sequential interaction of three electrons with the nuclear bath can generate a maximally entangled pair of electron spins. More generally, the class of states that can be generated via the spin bath is characterized in terms of matrix product states. Finally we show that imperfect electron spin operations, inhomogeneous couplings between electron and nuclei, and modifications to the ideal static spin bath still allow for the scheme to be realized. In situations where the spin-orbit coupling is large, our scheme can be an interesting alternative to the standard exchange based setups, because it does not involve occupation of any higher orbital levels.[16,17]

## II. ENTANGLEMENT GENERATION

We consider each electron coupled via the uniform Heisenberg interaction to the bath of $N$ nuclear spins and to an external magnetic field $B_z$ ($\hbar=1$),

$$H=\frac{A}{2N}(I^+S^-+S^+I^-)+\frac{A}{N}I^zS^z+g^*\mu_B B_z S^z. \tag{1}$$

$\mathbf{S}$ is the spin operator for the electron and $I^\mu=\Sigma_i I_i^\mu$ are the three components of the collective nuclear spin operators ($\mu=\pm,z$ and $[I^+,I^z]=-I^+$ and $[I^+,I^-]=2I^z$). $g^*$ is the electron $g$ factor and $\mu_B$ is the Bohr magneton. We consider spin-1/2 nuclei and neglect bath dynamics, the bath spins' Zeeman energies, and inhomogeneities in the Heisenberg couplings for now. We discuss the validity of these approximations toward the end of this paper.

We use the Dicke basis $\{|I,m,\beta\rangle\}$, where $I(I+1)$ is the eigenvalue of the collective angular momentum operator $\mathbf{I}^2$, the eigenvalue of $I^z$ is given by $m$, and $\beta$ is the permutation quantum number.[18] The initial state of the spin bath in the following is the identity

$$\rho_{\text{bath}}=\frac{1}{2^N}\sum_{I,m,\beta}|I,m,\beta\rangle\langle I,m,\beta|=\mathbb{1}_{2^N}/2^N. \tag{2}$$

In the following we omit $\beta$, which does not enter in the dynamics. This situation of a completely unknown bath state is, e.g., a suitable description for GaAs QDs even at temperatures as low as 100 mK.[9,10] In the following, time will be given in units of $N/A$. Even though the idea we present is applicable to any (quasi)static bath, we perform all estimations for GaAs, i.e., in particular $A^{-1}\approx 40$ ps.

The first electron spin (which we also refer to as ancilla

 

electron) is prepared in the state $|\uparrow\rangle$ and interacts *resonantly* for a time $t_1$ with the nuclear spin bath,

$$U|I,m,\uparrow\rangle = c_{Im}(t_1)|I,m,\uparrow\rangle + s_{Im}(t_1)|I,m+1,\downarrow\rangle, \tag{3}$$

with $U=e^{-iHt_1}$ and

$$c_{Im}(t_1) = \cos\left[\frac{(1+2I)t_1}{4}\right] - i\frac{1+2m}{1+2I}\sin\left[\frac{(1+2I)t_1}{4}\right],$$

$$s_{Im}(t_1) = \frac{-2i\sqrt{(I-m)(1+I+m)}}{1+2I}\sin\left[\frac{(1+2I)t_1}{4}\right].$$

Then the next electron spin, with initial state $|+\rangle = 1/\sqrt{2}(|\uparrow\rangle+|\downarrow\rangle)$, interacts for a time $t_2$ *off-resonantly* (e.g., in the presence of a large $B_z$) with the spin bath. For $g^*\mu_B B_z \gg A/\sqrt{N}$, the flip-flop part of the Hamiltonian can be approximately neglected,[19] yielding

$$V(t_2)|m,+\rangle = \frac{1}{\sqrt{2}}|m\rangle(e^{-i(\tilde{B}+m)t_2/2}|\uparrow\rangle + e^{+i(\tilde{B}+m)t_2/2}|\downarrow\rangle),$$

where $\tilde{B}=g^*\mu_B B_z N/A$ and the index $I$ has been omitted for brevity. Remarkably, by choosing the interaction time $t_2=\pi$, the state of the electron is transformed to $(-i)^m|(-)^m\rangle$, i.e., for even $m=2k$ to $(-1)^k|+\rangle$ and for odd $m=2k+1$ to $-i(-1)^k|-\rangle$. For convenience we assume that $\tilde{B}t_2/2=2\pi\ell$, $\ell\in\mathbb{N}$, which is adjusted by the "free" parameter of the large field. With the third electron, also in $|+\rangle$ initially and with the same interaction, the state becomes

$$\pm c_{Im}(t_1)|I,m,\uparrow\rangle|\pm\pm\rangle \mp s_{Im}(t_1)|I,m+1,\downarrow\rangle|\mp\mp\rangle, \tag{4}$$

with upper (lower) signs referring to even (odd) $m$.

In the final step, the ancilla electron interacts resonantly with the nuclei again [cf. Eq. (3)], giving

$$\pm c_{Im}(t_1)|\pm\pm\rangle[c_{Im}(t_1)|m,\uparrow\rangle + s_{Im}(t_1)|m+1,\downarrow\rangle],$$

$$\mp s_{Im}(t_1)|\mp\mp\rangle[c^*_{Im}(t_1)|m+1,\downarrow\rangle + s_{Im}(t_1)|m,\uparrow\rangle] \tag{5}$$

for even/odd $m$, and is eventually measured projectively in the $z$ basis. If the measurement outcome is $\downarrow$, it is clear from Eq. (5) that in each subspace the second and third electrons are in the maximally entangled state

$$|m+1\rangle(|++\rangle - e^{\pm i\phi_m}|--\rangle)/\sqrt{2}, \tag{6}$$

where the phase $\phi_m=2\arg(c_{Im})$ depends on the quantum numbers $I$ and $m$, leading to a washing out of the entanglement when the average over the different subspaces is taken. However, for short times $[(2I+1)t_1 \ll 1$ for typical values of $I\sim\sqrt{N}]$, this phase tends to zero and near ideal entanglement is created, albeit at the price of a lower success probability, see Fig. 1.

## III. MULTIPARTITE ENTANGLEMENT

The presented scheme generalizes in a straightforward manner to multipartite entanglement creation. Following the same protocol using $n$ electrons with arbitrary initial states $|\psi_1\rangle,\ldots,|\psi_n\rangle$, the final state becomes

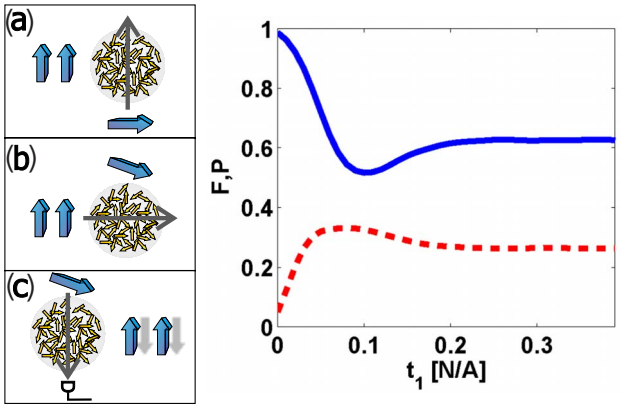


FIG. 1. (Color online) Left: Sketch of the protocol. (a) The $z$-polarized "control electron" interacts resonantly with the nuclear spin bath. (b) A sequence of $x$-polarized electrons interacts off-resonantly with the bath. (c) The control electron interacts resonantly again and is then measured in the $z$ basis. Right: Time dependence of overlap $F$ with Bell state $|\phi_-\rangle=(|++\rangle-|--\rangle)/\sqrt{2}$ (solid blue line) for $N=10^3$. The dashed red line shows the probability $P$ for a $\downarrow$ measurement.

$$|\Psi_n\rangle = \frac{1}{\sqrt{2}}\{\mathbb{1}+[(-1)^{m+1}i\sigma_z]^{\otimes n}\}|\psi_1,\ldots,\psi_n\rangle, \tag{7}$$

where the matrices are given in the standard $z$ basis and we assumed the short-time limit $t_1\to 0$ for clarity. If $|\psi_k\rangle=|+\rangle$ for all $k$, this is an $n$-partite Greenberger-Horne-Zeilinger (GHZ) state. The $m$-dependent relative phase in the above equation restricts to generation of GHZ states with even particle number.

When multiple resonant interactions with the ancilla and varying interaction times are allowed, a larger class of states becomes accessible. To see which states can in principle be prepared, we exploit the similarity of our setup to the sequential entanglement generation scheme analyzed in Ref. 20. There it was shown that all the matrix product states (MPSs) of bond dimension $d$ can be prepared if a string of qubits interacts sequentially with a $d$-dimensional ancilla system and *arbitrary unitaries* can be performed on ancilla and qubit in every step.

To apply this result to the present case, the ancilla electron and nuclear spin system together represent the control qubit: an effective $d=2$ system with Hilbert space spanned for given $(I,m)$ by $\{|I,m,\uparrow\rangle,|I,m+1,\downarrow\rangle\}$. To see that arbitrary unitaries are possible, note that $x$ rotations of the control qubit are caused by resonant interaction, while a static $B_z$ field causes $z$ rotations. From these, all single-qubit gates on the control qubit can be constructed. The off-resonant interaction considered before performs essentially a CNOT gate between the passing and the control qubit. In the CNOT gate the "control qubit" is the control and the passing electron is the target, in the $|\uparrow,\downarrow\rangle$ and $|\pm\rangle$ basis, respectively. Combined with single-qubit gates (on the passing electron), this seems to be enough to allow for arbitrary transformations on the coupled control-target system. However, the situation is more complicated since the effective gate performed by the off-resonant interaction *differs* for even and odd parities of the control qubit, namely, $V(\pi)=e^{(-1)^m i(\pi/4)\sigma_z}\sigma^m_{x,1}CNOT_{1\to 2}\sigma^m_{x,1}$. I.e., not only there is, as seen

before, a parity-dependent phase but also whether logical-0 or logical-1 controls the bit flip in the passing qubit depends on the parity of $m$. One way to remove this $m$-dependence and enable the generation of arbitrary states is to perform an "$I^z$ parity measurement" by sending an electron $|+\rangle$ into the dot, and then measure it in the $|\pm\rangle$ basis after off-resonant interaction for a time $\pi$. Depending on the outcome, either the odd or the even states are projected out. Remarkably, gaining this single bit of information about the $2^N$-dimensional bath then allows us to remove all $m$-dependences and perform clean CNOT gates. Hence the interactions outlined above are sufficient to prepare all $d=2$ MPSs with high fidelity. If the passing electrons can be brought into interaction with the ancilla again at any time, *arbitrary* two-qubit gates can be performed, which implies that all matrix product states with two-dimensional bonds can be sequentially created.[20]

Direct resonant interactions lead to very low-fidelity $x$ rotations due to averaging over the different subspaces, indicating that prior measurement[21–23] or cooling[24] of the spin bath might be necessary. More sophisticated control schemes, however, allow for near-unit-fidelity single-qubit rotations with no prior preparation: In Ref. 25 it was proven that high-fidelity arbitrary single-qubit gates can be effected by a Hamiltonian $H=\delta\sigma_z+\Omega(\sigma_x\cos\phi+\sigma_y\sin\phi)$, when only the parameter $\phi$ can be controlled precisely. For $\delta$ and $\Omega$ it is sufficient to know that they are nonzero for some value of a controllable external parameter and zero for another. In our situation we have the three Hamiltonians $H_1=\Delta\sigma_z=B\mu_I\sigma_z/2$ (nuclear Zeeman), $H_2=\frac{A}{2N}[(m+1/2)\sigma_z+\xi_{I,m}\sigma_x]$ (resonant HF), and $H_3=\frac{A}{2N}[\pm(\tilde{B}+m+1/2)+\tilde{B}\mu_I/(\mu_B g^*)]\sigma_z$ (off-resonant HF) at hand. The Pauli matrices act on the control qubit, $\mu_I$ is the nuclear magnetic moment, and $\xi_{I,m}=\sqrt{I(I+1)-m(m+1)}$. The plus and minus signs for $H_3$ can be effected through spin flips of the passing electron (recall that $\sigma_x e^{iHt}\sigma_x=e^{i\sigma_x H\sigma_x t}$ and $\sigma_x\sigma_z\sigma_x=-\sigma_z$). These Hamiltonians can be switched on and off (adiabatically[26]) at will. Appropriate iterations of evolutions can lead to effective Hamiltonians of weighted sums and commutators of $H_{1,2,3}$. In particular, the subspace independence of the parameter $\Delta\propto B$ allows for generation of any weighted sum of $\sigma_x$ and $\sigma_y$ with the weights being $(I,m)$ independent, thus making the results of Ref. 25 applicable. We have thus shown that while naive use of resonant interactions will lead to poor gate fidelities for the control qubit, enhanced control schemes still allow for full access to high-fidelity rotations. Hence, in principle, universal quantum computation on an electron spin quantum register can be performed, with all interactions mediated by the highly mixed spin bath.

## IV. EXPERIMENTAL FEASIBILITY

We discuss now various couplings that have been neglected in the idealized Hamiltonian (1) but are present in the QD setup. We are concerned here only with their effects on the basic entanglement generation scheme. It is clear that the scheme can only work as long as it is fast compared to the electron $T_2$ time, since the coherence of the ancilla electron must be preserved. We see below that neither nuclear dynamics nor inhomogeneity place more stringent conditions on our scheme.

### A. Inhomogeneity

The HF Hamiltonian in QDs has a slightly different form from the one in Eq. (1), because the collective bath operators have a spatial dependence $A/NI^\mu\to A^\mu\equiv\Sigma_i\alpha_i I_i^\mu$, with $\mu=\pm,z$. The coupling constants $\alpha_i$ are $\propto\mu_{I,i}|\psi_e(r_i)|^2$, with $|\psi_e(r_i)|^2$ being the probability of finding the electron at location $r_i$, and $A=\Sigma_j\alpha_j$ denotes now the effective (average) hyperfine coupling strength. We focus our analysis on short resonant interaction times $\Delta t_1\ll\sqrt{N}/A$. The electronic state after the above protocol conditioned on a $\downarrow$ measurement is proportional to

$$\sum_{\psi,\psi'}\langle\psi'|A^+e^{i(\sigma_1^z+\sigma_2^z)A^z t_2}+e^{i(\sigma_1^z+\sigma_2^z)A^z t_2}A^+|\psi\rangle|++\rangle\times\text{H.c.},$$

where the Pauli matrices act on the off-resonant electrons and $|\psi\rangle=|i_1\cdots i_N\rangle$ and $|\psi'\rangle=|i_1'\cdots i_N'\rangle$ label the orthonormal basis of $I_j^z$ eigenstates. Evaluating the matrix elements and introducing the normalization, we get

$$\rho(t_2)=\frac{1}{\mathcal{N}(t_2)}\sum_j\alpha_j^2\sum_{\substack{i_1,\ldots,i_N=\pm1/2\\ i_j=-1/2}}(|+_0+_0\rangle+|-_j-_j\rangle)\times\text{H.c.},\quad(8)$$

with the states $|+_0(t_2)\rangle=e^{i\omega_0 t_2/2}|\uparrow\rangle+e^{-i\omega_0 t_2/2}|\downarrow\rangle$ and $|-_j(t_2)\rangle=e^{i\omega_j t_2/2}|\uparrow\rangle+e^{-i\omega_j t_2/2}|\downarrow\rangle$, both of which depend on the nuclear spin configuration $\{i\}$ via the frequencies $\omega_0=\omega_0(\{i\})=\Sigma_\ell\alpha_\ell i_\ell$ and $\omega_j=\omega_0+\alpha_j$ and the normalization $\mathcal{N}(t_2)=\Sigma_j\alpha_j^2[3+\cos(\alpha_j t_2)]$. The time dependence of the states has been omitted for brevity in the above formula. Straightforwardly, one now determines the fidelity $F(t_2)=\langle\phi_-|\rho(t_2)|\phi_-\rangle$ with the desired maximally entangled state $|\phi_-\rangle\sim|+_0+_0\rangle+|-_0-_0\rangle$ as

$$F(t_2)=2\sum\alpha_j^2\Big/\sum_j\alpha_j^2[3+\cos(\alpha_j t_2)].\quad(9)$$

This expression readily gives the fidelity for arbitrary particle numbers and arbitrary distributions of coupling constants. For $N\gg1$, the obtained value is independent of particle number, and for the relevant situation of Gaussian coupling $F=0.90$, 0.83, and 0.78 for one dimension, two dimensions, and three dimensions, respectively. Including the difference in magnetic moments for Ga and As [$^{75}$As: $\mu_{I,\text{As}}=1.44$; $^{69}$Ga: $\mu_{I,\text{Ga},1}=2.02$ (60%); and $^{71}$Ga: $\mu_{I,\text{Ga},2}=2.56$ (40%) (Ref. 7)], these values become $F=0.83$, 0.78, and 0.74, indicating that our scheme is not compromised by realistic inhomogeneities.

For small inhomogeneity we find the optimal time $t_2^{(\text{opt})}$ by setting the time derivative of $F$ zero and expanding the equation in terms of the deviations $\epsilon_j=\alpha_j-\alpha^*$, where $\alpha^*=\pi/t_2^{(\text{opt})}$. Going to second order in the small parameters $\epsilon_j/\alpha^*$, the ensuing quadratic equation yields $\alpha^*=\eta_1[5-\sqrt{1+24(1-\eta_2/\eta_1^2)}]/4$, with $\eta_x=\frac{1}{N}\Sigma_j\alpha_j^x$. Plugging $t_2^{(\text{opt})}$ back into Eq. (9) and keeping terms up to second order, we find $F(\pi/\alpha^*)=1-\frac{\pi}{2N}\Sigma_j(\epsilon_j/\alpha^*)^2$.

### B. Nuclear Zeeman energies

For the times considered, nuclear Zeeman energies lead to an important relative phase $B_z\mu_{I,j}t_2$ for each of the two terms in the sum of the conditional state given in Eq. (8). Considering one homogeneously coupled species of nuclear spins, the state of Eq. (6) will have an additional $m$-dependent phase. In each invariant subspace this produces an overall phase $\propto B_z\mu_I t_2 m$ and a relative phase $\propto B_z\mu_I t_2$ between the two parts of the superposition. This might not seem harmful, but due to the parity effect, the *sign* of the phase depends on the parity of $m$. Since this phase is of order $\pi$, it could spoil the protocol. However, by simply waiting for an appropriate time $t_p$ after each of the $n$ electrons has passed, the total relevant phase is $(-1)^m nB_z\mu_I(t_2+t_p)$; with $t_p+t_2$ an integer multiple of $\pi/(\mu_I B_z)$, it is again $m$ independent. By the same procedure, the nuclear Zeeman related phase can be removed for single-species inhomogeneous systems.

For systems with strongly varying nuclear magnetic moments $\mu_{I,j}$, the relative phase depends on "which nuclear spin has flipped" and the waiting time needs to be chosen such that all the relative phases are close to $2k\pi$. Otherwise the final fidelity may be strongly degraded. For the three species in GaAs this is the case, e.g., for $B_z(t_2+t_p)\approx 7\pi$, and assuming that a flat wave function still allows for a fidelity $\gtrsim 0.9$ with moderate overhead in time. In principle, one can completely cancel the undesired phase by removing the electrons from the QD and reversing the magnetic field for $t_p=t_2$.

### C. Bath dynamics

The major internal dynamics of nuclear spins in QDs stems from the indirect hyperfine mediated interaction and the direct dipolar interaction.[6,8] Both mechanisms lead to bilocal errors that contain spin-flip terms $\propto(\Gamma_d^{kl}+\Gamma_i^{kl})I_k^+I_l^-$ and phase changing $zz$ terms $\propto\Gamma_d^{kl}I_k^zI_l^z$. The transition rates for direct and indirect interactions are $\Gamma_{dd}/r_{kl}^3$ and $\alpha_i\alpha_j/\Omega_e$, respectively, where $\Omega_e$ is the electron Zeeman energy and $r_{kl}=|\mathbf{r}_k-\mathbf{r}_l|$.

The dephasing interactions $\propto I_i^zI_j^z$ lead to a relative phase between the terms in Eq. (8), similar to the nuclear Zeeman energies. The energy difference, i.e., (in a mean-field treatment) the Zeeman splitting of a single nuclear spin in the field of its neighbors, is a few times $\Gamma_{dd}$.[24] Thus we need $N\Gamma_{dd}/(r_0^3A)\ll 1$; given $\Gamma_{dd}/r_0^3\approx 0.1$ ms (Ref. 7) for nearest neighbors and $A\approx 40$ ps, this condition is readily fulfilled even for large dots.

We have seen above that for each term in the mixed state, the qubits rotate on the equatorial plane of the Bloch sphere with frequencies $\alpha_j$ when the $j$th nuclear spin has been flipped. If this particular spin is involved in a spin flip due to bath dynamics, the resulting rotation with "wrong" frequency spoils the entanglement. The errors in the rotation angle for the term containing the flip of the $j$th spin are $\epsilon_{d,i}^j=\sqrt{\Sigma_k(\Gamma_{d,i}^{kj}t)^2(\alpha_j-\alpha_k)^2(t_2^{\mathrm{opt}})^2}$, and the final overall errors are $\Sigma_j\alpha_j^2\epsilon_{d,i}^j/\Sigma_j\alpha_j^2$. We evaluate the above sums in the continuum limit for Gaussian couplings and get for the indirect flips a total error of $\delta_i\pi^2A\gamma_3^2/(\gamma_4^2\Omega)$, where $\delta_i$ is determined by the integrals over the coupling constants. Taking $A/\Omega<1$ for the large ($>1$ T) fields that we require, we find errors of 2.4%, 2.0%, and 1.5% for one dimension, two dimensions, and three dimensions, respectively, for $N=10^4$ (we define $N$ here as the number of nuclei within the $1/(2e)$ width of the Gaussian). For the direct nuclear dipole-dipole transitions, the error is of size $\delta_d\pi^2\gamma_3^2\Gamma_{dd}/(Ar_0^3\gamma_4^2)$, where numerical evaluation of the "dipolar integrals" $\delta_d$ yields 0.01%, 0.8%, and 5% for the same situation as above. This overall error is thus on the order of a few percent for realistic situations.

### D. Storage

We implicitly assumed the possibility of storing the electrons protected from any bath. In QD structures this could be achieved by shuttling the electrons to a nuclear spin-free region or employing dynamical decoupling schemes; see, for example, Ref. 6. The required storage times of a few tens of microseconds should be readily achieved.

### E. Imperfect electronic operations

A finite probability that an up electron is wrongly detected as a down electron (or vice versa) degrades the final entanglement. However, as only one electron needs to be measured, the effect is no worse for the $n$-partite GHZ state than for the Bell state $|\Phi^+\rangle$. The same goes for variations in the resonant interaction time $t_1$. In contrast, errors in the electron preparation and variations in the off-resonant interaction time $t_2$, since they affect each of the $n$ electrons, lead to a fidelity reduction that scales exponentially with $n$. Variations in $t_2$ must be such that $\tilde{B}\delta t_2\ll 1$, with $\tilde{B}\gg A/\sqrt{N}$, which makes this the most stringent but still realistic[27,28] requirement for electron timing.

## V. SUMMARY AND CONCLUSIONS

We have considered the Heisenberg interaction of electron spin qubits with a long-lived spin bath in a situation where *nothing* is known about the state of the bath. We have shown that nevertheless high-fidelity multipartite entanglement can be created via this bath.

The qubits interact neither directly with each other nor simultaneously with the bath at any time. Our protocol thus demonstrates that even the interaction with infinite temperature systems can mediate highly coherent operations and thus represent a valuable resource for quantum information processing that merits further investigation. In fact, when only one bit of information is extracted from the spin bath, arbitrary gates between the bath and the qubits are possible, and all matrix product states with two-dimensional bonds can be created by sequential interaction. The explicit protocols we presented can be realized in quantum dot setups and would (in typical GaAs dots) allow for the creation of entanglement between two electrons on a time scale of a few microseconds.

## ACKNOWLEDGMENTS

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